STUDENT BOOK ANSWERS

Chapter 6 Einstein's special relativity

Question set 6.1

1 a Roemer demonstrated that light has a finite speed, not an infinite speed as some people thought.

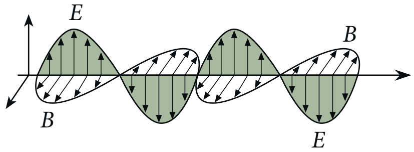
b Bradley also demonstrated the finite speed of light by measuring stellar aberration, and measured its speed to be $2.98 \times 10^8\,\mathrm{m\,s^{-1}}$.

c Young demonstrated the wave nature of light by performing interference experiments.

2 The speed of electromagnetic radiation depends on two properties of the material it travels through, the electrical permittivity, ε_0, and the magnetic permeability, μ_0, $v = \dfrac{1}{\sqrt{\mu_0 \varepsilon_0}}$.

3 Maxwell combined all the existing theories of electromagnetism into 4 equations that describe all electromagnetic phenomena. He unified the theories of electricity and magnetism by giving the mathematical relationship between them.

4



E is the electric field that oscillates as a transverse wave travelling in the direction of the arrow showing the direction of motion. B is the magnetic field also oscillating as a transverse wave but perpendicular to the direction of the E field. The direction of propagation (the velocity vector) is perpendicular to both fields in a vacuum.

5 Hertz used a high-voltage spark gap to produce electromagnetic waves that were detected a few metres away. He showed that these waves could be reflected and refracted, and had a speed of $3.0 \times 10^8\,\mathrm{m\,s^{-1}}$, and hence that these waves had the properties of light. This was experimental confirmation of predictions based on Maxwell's equations and was important in Maxwell's equations gaining acceptance.

6 Electromagnetic waves can be used to accelerate electrons because both electric and magnetic fields exert forces on electrons and other charged particles. An electric field exerts a force on any charged particle, and a magnetic field exerts a force on any moving charged particle. This means that if there is no other force preventing the electrons from moving, they will accelerate. For example, when an electromagnetic wave is incident on a metal, the free electrons are accelerated in the direction of the electric field. This is how antennae work, as described in Chapter 5.

7 Students' answers will vary. They should discuss the interplay between theory and experiment, using Maxwell's theory of electromagnetism and the experiments that supported it and led to its development as an example.

Worked example 6.1

Try these yourself

a

$v = 30\,\mathrm{m\,s^{-1}} + 2\,\mathrm{m\,s^{-1}} = 32\,\mathrm{m\,s^{-1}}$

$x = x' + v\Delta t$

$\Rightarrow x = 0 + (32\,\mathrm{m\,s^{-1}})(5.0\,\mathrm{s})$

$\Rightarrow x = 160\,\mathrm{m}$

Logic	Marks
Recognise that the person moves at a total speed of $30\,\mathrm{m\,s^{-1}} + 2\,\mathrm{m\,s^{-1}}$ relative to the station, so $v = 32\,\mathrm{m\,s^{-1}}$ in this frame.	
Relate position to speed.	1 mark
Correct substitution with units.	1 mark
Correct answer.	1 mark

b

$v = 2\,\mathrm{m\,s^{-1}}$

$x = x' + v\Delta t$

$x = 0 + (2.0\,\mathrm{m\,s^{-1}})(5.0\ \mathrm{s})$

$x = 10\,\mathrm{m}$

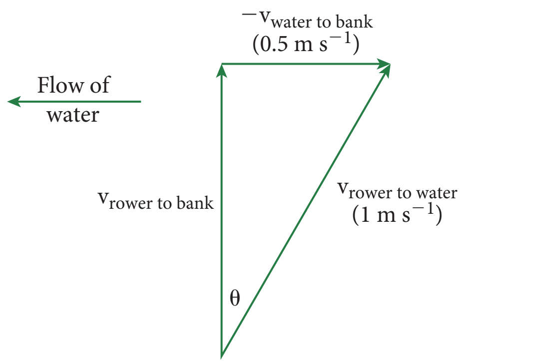
Logic	Marks
Recognise that the person is moving at only $2.0\,\mathrm{m\,s^{-1}}$ with respect to the train's reference frame.	
Relate position to speed.	1 mark
Correct substitution with units.	1 mark
Correct answer.	1 mark

Worked example 6.2

Try these yourself

a

$$\mathrm{v_{rower\ to\ water}} = \mathrm{v_{rower\ to\ bank}} + \mathrm{v_{bank\ to\ water}}$$
$$= \mathrm{v_{rower\ to\ bank}} + -\mathrm{v_{water\ to\ bank}}$$



3 marks

b

The velocity is perpendicular to the bank

$v_{\text{boat to water2}} = v_{\text{boat to bank2}} + v_{\text{water to bank2}}$

$v_{\text{boat to bank}} = (v_{\text{boat to water2}} - v_{\text{water to bank2}})^{1/2}$

$v_{\text{boat to bank}} = [(1.0\,\mathrm{m\,s^{-1}})^2 - (0.5\,\mathrm{m\ s^{-1}})^2]^{1/2}$

$v_{\text{boat to bank}} = 0.87\,\mathrm{m\,s^{-1}}$

Logic	Marks
(Given in the question)	1 mark
Use Pythagorean formula to relate the relative velocities.	1 mark
Rearrange for v.	1 mark
Substitute values with units.	1 mark

Question set 6.2

1 An inertial reference frame is one that is not accelerating. We can tell that a frame is not accelerating because Newton's first law is obeyed in such a frame.

2 $x = x' + v\Delta t$

$y = y' + v\Delta t$

$\vec{v}_{AB} = \vec{v}_{AC} + \vec{v}_{CB}$

3 In the reference frame of the ship the object is released from a height with no horizontal velocity. The only force acting on it is gravity, so it follows a path directly downwards.

In the reference frame of someone stationary relative to Earth's surface, the object is released with a horizontal component to its velocity equal to the velocity of the ship. The only force acting on the object once it is released is gravity, which is perpendicular to the object's horizontal velocity. Hence it has an acceleration downwards due to gravity, and a constant horizontal velocity. This is exactly what was described in Chapter 1 as projectile motion. The path in this case is a parabola.

4 If you are in an inertial reference frame then Newton's first and second laws are obeyed. Any object will continue in its state of motion unless a force acts upon it. If you see objects accelerating with no force acting upon them, then you are not in an inertial reference frame. A common example is a curved trajectory with no force – you may observe this in a rotating frame such as on a merry-go-round. Another common example is the apparent change in your weight in an accelerating lift, as described in Chapter 2.

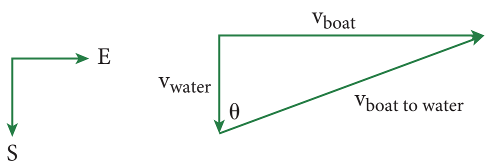
5 a i $v_{\text{person to bus}} = 2.0\,\text{m s}^{-1}$

ii $v_{\text{person to outside}} = v_{\text{person to bus}} + v_{\text{bus to outside}} = 2.0\,\text{m s}^{-1} + 12.0\,\text{m s}^{-1} = 14\,\text{m s}^{-1}$

b i $x = v\Delta t = (2.0\,\text{m s}^{-1})(5.0\,\text{s}) = 10\,\text{m}$

ii $x = v\Delta t = (14.0\,\text{m s}^{-1})(5.0\,\text{s}) = 70\,\text{m}$

6



$v_{\text{boat to water}} = \sqrt{v_{\text{boat}}^{2} + v_{\text{water}}^{2}}$

$\Rightarrow v_{\text{boat to water}} = \sqrt{(4.0\text{ m s}^{-1})^2 + (3.0\text{ m s}^{-1})^2} = 5.0\text{ m s}^{-1}$

$\tan\theta = \dfrac{4.0\text{ m s}^{-1}}{3.0\text{ m s}^{-1}}$

$\Rightarrow \theta = 53°$

$\Rightarrow v_{\text{boat to water}} = 5.0\text{ m s}^{-1}$ N53°E

7 Two thought experiments from the Dialogues are described.

In the first of Galilieo's thought experiment described in the chapter, a sailor drops an object from the mast of a sailing ship moving at steady velocity. He asked the question: 'Where would the object land relative to the deck of the ship?' In his frame of reference the sailor would see the object fall straight down parallel to the mast; however, a nearby observer who is on land would see from his frame of reference that the object would follow a parabolic path. This is because the acceleration is the same in both reference frames although the velocity is not. In particular, the object has no horizontal velocity in the ship's reference frame, but it does in the reference frame of a nearby observer.

In the second thought experiment, Galileo imagined a person walking within the cabin of a sailing ship. If the sailing ship moves forwards at a velocity of $5\,m\,s^{-1}$, a person moving forwards at a velocity of $1\,m\,s^{-1}$ relative to the cabin will be moving forwards at a velocity of $6\,m\,s^{-1}$ relative to Earth. The relative position and velocity of the person is different in each frame of reference.

8 Students' answers will vary, but should include the Galilean position and velocity transformations and the idea of inertial reference frames.

Question set 6.3

1 The aether is the hypothetical medium through which light travels, which remains at rest relative to Earth and Sun. It was hypothesised that light is a vibration of the aether in the same way that sound is a vibration of air or other medium.

2 The aim of the Michelson–Morley experiment was to detect the aether by looking for changes in the interference of light as their interferometer was rotated. They did not detect any change in the interference pattern, and hence they did not find evidence for the existence of the aether.

3 They used light because they were attempting to detect the aether – the medium through which light was supposed to travel. If the light was moving relative to the medium, then changing the direction of the light's path relative to the medium should have resulted in a relative increase or decrease in the speed of light.

4 For both a and b, the movement of Earth's surface and hence their equipment relative to the aether should result in a change in the interference pattern when measurements are repeated with a time delay. For a, taking measurements some months apart, when Earth is travelling in a different direction relative to the aether should result in a shift in the interference pattern. For b, taking measurements some hours apart would give different patterns. See Figure 6.10.

5 A shift in the interference pattern would have given evidence for (but not actually confirmed the existence of) the aether, because it implies a medium through which light propagates, which means there is a single reference frame in which light is moving at c. In all other frames, which are moving relative to this medium, light will have a different speed. If light had a different speed when our frame of reference (Earth) is moving in different directions, then we should be able to observe the effects of the varying speed of light at different times of day or year as shifts in the interference pattern.

6 Students' answers will vary, but should mention the interplay between theory and experiment, and the central role of experiment in testing theories. They should also mention the importance of falsifiability – the aether theory was discarded because its predictions were not observed when the experiments were done.

Worked example 6.4

Try these yourself

a

	Logic	Marks
$L' = \frac{L_o}{\gamma} = L_o\sqrt{1 - \frac{v^2}{c^2}}$	Relate L' to data given.	1 mark
$L' = 4.4\ \text{ly}\sqrt{1 - 0.4^2}$ $\Rightarrow L' = 4.0\ \text{ly}$	Substitute values and calculate final answer.	1 mark

b

	Logic	Marks
$t_{Earth} = \frac{L_o}{v}$ $\Rightarrow t_{Earth} = \frac{4.4\ \text{ly}}{0.4\ \text{c}}$ $\Rightarrow t_{Earth} = 11\ \text{ly}$	Calculate t_{Earth}.	
$t' = \frac{L'}{v}$ $\Rightarrow t' = \frac{4.0\ \text{ly}}{0.4\ \text{c}}$ $\Rightarrow t' = 10\ \text{ly}$	Calculate t′.	1 mark
$\Delta t = t_{Earth} - t'$ $\Rightarrow \Delta \text{t} = 11\ \text{ly} - 10\ \text{ly} = 1\ \text{ly}$	Calculate Δt.	1 mark

Worked example 6.5

Try these yourself

a **i** $t = \frac{s}{v}$

$$\Rightarrow t = \frac{3.0 \times 10^3\ \text{m}}{0.996 \times 3.0 \times 10^8\ \text{m s}^{-1}}$$

$$\Rightarrow t = 10\ \mu\text{s}$$ 1 mark

ii $n = \frac{t}{t_{mean}}$

$$\Rightarrow n = \frac{10\ \text{mu s}}{2.2\ \text{mu s}}$$

$$\Rightarrow n = 4.5\ \text{mean life times}$$ 1 mark

iii The time taken is only a few times the mean lifetime of a muon, so a small fraction will make it to the ground. Remember that the mean life time is only an average, many have longer (and shorter) lifetimes. 1 mark

b $n = \frac{t}{t_{\text{mean}}}$,Earth $= \frac{t}{\gamma t_{\text{mean}}}, = \mu = \frac{n}{\gamma} = \sqrt{1 - \frac{v^2}{c^2}} = 4.6\sqrt{1 - 0.996^2} = 0.41$ 1 mark

Worked example 6.6

Try this yourself

	Logic	Marks
$L' = \frac{L_o}{\gamma}$ $\Rightarrow L' = L_o\sqrt{1 - \frac{v^2}{c^2}}$	Relate L' to data given.	1 mark
$L' = 8.0\text{ m}\sqrt{1 = 0.9^2}$ $\Rightarrow L' = 3.5\text{ m}$	Substitute values and calculate L'.	1 mark
$\Delta L = L' - L_o = 8.0\text{ m} - 3.5\text{ m} = 4.5\text{ m}$	Calculate ΔL.	1 mark

Question set 6.4

1 **a** Time measured in the rest frame of the object being measured, that is in the frame in which the object is not moving.

b The length measured in the rest frame of the object being measured.

2 The laws of physics are the same in all inertial frames.

The speed of light, *c*, is constant in all inertial reference frames.

3 Simultaneity is the idea that two events that occur at the same time in one frame will also be seen to occur at the same time in different reference frames – this is not in fact the case. Events that are simultaneous in one frame may not be simultaneous in another frame.

4 All of the conservation laws (energy, momentum, charge, etc) are true in all inertial reference frames.

5 **a** Time appears slower in a reference frame that is moving relative to a ‘stationary’ frame.

b Length appears shorter in a reference frame that is moving relative to a ‘stationary’ frame.

6 $L' = \frac{L_o}{\gamma}$

$\Rightarrow L' = L_o\sqrt{1 - \frac{v^2}{c^2}}$

$\Rightarrow L' = 50\text{ m}\sqrt{1 - 0.6^2}$

$\Rightarrow L' = 40\text{ m}$

7 The ends of the carriage reaching the markers are simultaneous events in the reference frame of someone outside the train. They are not simultaneous to someone travelling on the train – so in their reference frame the carriage does not fit exactly between the markers.

8 $$t_{\text{pion}} = \frac{t_{\text{Earth}}}{\gamma}$$

$$\Rightarrow t_{\text{pion}} = t_{\text{Earth}}\sqrt{1 - \frac{v^2}{c^2}}$$

$$\Rightarrow t_{\text{pion}} = 26 \text{ ns}\sqrt{1 - 0.75^2}$$

$$\Rightarrow t_{\text{pion}} = 17 \text{ ns}$$

9 Students' answers will vary.

10 Students' answers will vary.

Worked example 6.7

Try these yourself

1

	Logic	Marks
$m = \gamma m_o$ $\Rightarrow m = \dfrac{m_o}{\sqrt{1 - \frac{v^2}{c^2}}}$	Relate m to m^o.	1 mark
$m = \dfrac{1.7 \times 10^{-27} \text{ kg}}{\sqrt{1 - 0.75^2}}$ $\Rightarrow m = 2.6 \times 10^{-27} \text{ kg}$	Substitute values and calculate m.	1 mark

2

	Logic	Marks
$m = \gamma m_o$ $\Rightarrow m = \dfrac{m_o}{\sqrt{1 - \frac{v^2}{c^2}}}$	Relate m to m^o.	1 mark
$\dfrac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = \dfrac{m}{m_o} = 2.5$	Relate m and m^o to γ.	1 mark
$v = c\sqrt{1 - \dfrac{m_o^2}{m^2}}$	Rearrange for v.	1 mark
$v = c\sqrt{1 - \dfrac{1}{2.5^2}} = 0.92\,\text{c}$	Substitute and calculate v .	1 mark

Scientific literacy: Energy and momentum at the Australian Synchrotron

1 **a** 3 GeV

b Approximately 35 m

c 0.99999995 c

d 1.3 T

e $\gamma = \dfrac{1}{\sqrt{1 - \dfrac{v^2}{c^2}}}$

i at 0.1 c : $\gamma = \dfrac{1}{\sqrt{1 - 0.1^2}} = 1.005$

ii at 0.999 999 95 c: $\gamma = \dfrac{1}{\sqrt{1 - 0.999\,999\,95^2}} = 3162$

2 **a** $\beta = \dfrac{v}{c} = 0.999\,999\,95$

b $v = 0.999\,999\,95\,c = 2.997\,999\,9\ \text{m s}^{-1}$

3 Classically:

$p = mv = 9.11 \times 10^{-31}\ \text{kg} \times 0.999\,999\,95 \times 3.00\ 10^{8}\ \text{m s}^{-1} = 2.73 \times 10^{-22}\ \text{kg m s}^{-1}$

$KE = \dfrac{1}{2}mv^2 = \dfrac{1}{2} \times 9.11 \times 10^{-31}\ \text{kg} \times (0.999\,999\,95 \times 3.00 \times 10^{8}\ \text{m s}^{-1})^2 = 4.10 \times 10^{-14}\ \text{J}$

Relativistically:

$$p = \gamma p_o = \gamma m_o v = \frac{m_o v}{\sqrt{1 - \dfrac{v^2}{c^2}}}$$

$$= \frac{9.1 \times 10^{-31}\ \text{kg} \times 0.9996 \times 2.998 \times 10^{8}\ \text{ms}^{-1}}{\sqrt{1 - 0.999\,999\,95^2}} = 8.6 \times 10^{-19}\ \text{kg m s}^{-1}$$

$$KE = (\gamma - 1)\, m_o c^2 = \left(\frac{1}{\sqrt{1 - \dfrac{v^2}{c^2}}} - 1 \right) m_o c^2$$

$$= \left(\frac{1}{\sqrt{1 - 0.999\,999\,95^2}} - 1 \right) (9.11 \times 10^{-31}\ \text{kg}) \times (3.00 \times 10^{8}\ \text{m s}^{-1})^2 = 2.6 \times 10^{-10}\ \text{J}$$

Complete the table:

	Kinetic energy (J)	Momentum (N s)
Classical	4.1×10^{-14}	2.7×10^{-22}
Relativistic	2.6×10^{-10}	8.6×10^{-19}

$\mathbf{b} = \frac{v}{c}$, and so $v = \beta c$

Classically: $E_k = \frac{1}{2} mv^2 = \frac{1}{2} m (\beta c)^2$

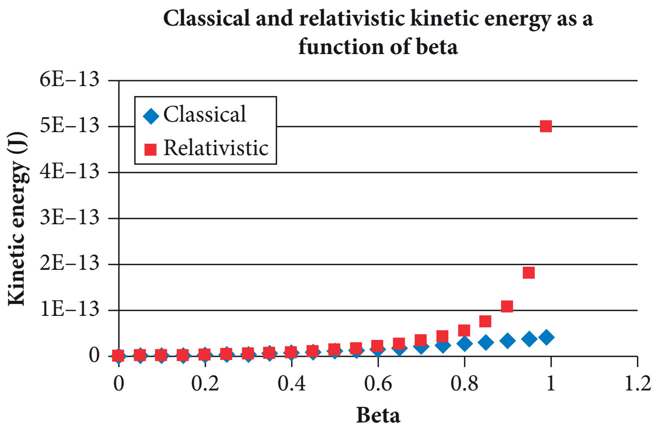
Relativistically: $KE = (\gamma - 1)\, m_o c^2 = \left(\frac{1}{\sqrt{1-\beta^2}} - 1 \right) m_o c^2$

$1000\,\text{keV} = 1.6 \times 10^{-13}\,\text{J}$

Using a spreadsheet with these equations, we can produce the following table:

Beta	v	Classical	Gamma	Relativistic
0	0	0	1	0
0.05	15 000 000	1.02E−16	1.001252	1.0268E−16
0.1	30 000 000	4.10E−16	1.005038	4.1305E−16
0.15	45 000 000	9.22E−16	1.011443	9.3825E−16
0.2	60 000 000	1.64E−15	1.020621	1.6907E−15
0.25	75 000 000	2.56E−15	1.032796	2.6889E−15
0.3	90 000 000	3.69E−15	1.048285	3.9589E−15
0.35	1.05E+08	5.02E−15	1.067521	5.5360E−15
0.4	1.20E+08	6.56E−15	1.091089	7.4684E−15
0.45	1.35E+08	8.30E−15	1.119785	9.8212E−15
0.5	1.50E+08	1.02E−14	1.154701	1.2684E−14
0.55	1.65E+08	1.24E−14	1.197369	1.6182E−14
0.6	1.80E+08	1.48E−14	1.25	2.0498E−14
0.65	1.95E+08	1.73E−14	1.315903	2.5901E−14
0.7	2.10E+08	2.01E−14	1.40028	3.2819E−14
0.75	2.25E+08	2.31E−14	1.511858	4.1967E−14
0.8	2.40E+08	2.62E−14	1.666667	5.4660E−14
0.85	2.55E+08	2.96E−14	1.898316	7.3653E−14
0.9	2.70E+08	3.32E−14	2.294157	1.0611E−13
0.95	2.85E+08	3.70E−14	3.202563	1.8059E−13
0.99	2.97E+08	4.02E−14	7.088812	4.9922E−13
0.995	2.99E+08	4.06E−14	10.01252	7.3894E−13

And graph:



4 a i The article says it is approximately 10 cm.

ii $\lambda = \frac{\lambda_o}{\gamma} = \lambda_o \sqrt{1 - \frac{v^2}{c^2}} = 0.1\,\text{m}\sqrt{1 - 0.999\,999\,95^2} = 3 \times 10^{-5}\,\text{m}$

iii $\lambda_{obs} = \lambda_{em}\sqrt{\frac{1 - \frac{v}{c}}{1 + \frac{v}{c}}} = 3 \times 10^{-5}\,\text{m}\sqrt{\frac{1 - 0.999\,999\,95}{1 + 0.999\,999\,95}} = 5 \times 10^{-9}\,\text{m or } 5\,\text{nm}$

b The calculated wavelength is in the nm range, so it does agree with the claim in the article.

5 In the laboratory frame of reference:

$p = qBr = \gamma m_o v$

so $r = \frac{\gamma m_o v}{qB} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}\left(\frac{m_o v}{qB}\right)$

$= \frac{1}{\sqrt{1 - 0.999\,999\,95^2}}\left[\frac{9.11 \times 10^{-31}\,\text{kg} \times 0.999\,999\,95 \times 3.00 \times 10^8\,\text{m s}^{-1}}{1.6 \times 10^{-19}\,\text{C} \times 1.3\,\text{T}}\right]$

$= 4.2\,\text{m}$

This is wider than a kitchen table.

In the electrons frame:

$r = \frac{r_o}{\gamma} = r_o\sqrt{1 - \frac{v^2}{c^2}} = 4.2\,\text{m}\sqrt{1 - 0.999\,999\,95^2} = 1.3 \times 10^{-3}\,\text{m}$

This would easily fit on a kitchen table, or into your pocket.

6 a Students' answers will vary.

b Students' answers will vary.

Worked example 6.9

Try this yourself

	Logic	Marks
$m_{U238} = 238.050\,788\,u$ $m_{He4} = 4.002\,603\,u$ $m_{Th234} = 234.040\,952\,u$	Look up the masses.	2 marks
$\Delta m = m_{U238} - m_{Th234} - m_{He4}$	Write the expression for the mass change.	1 mark
$\Delta m = 238.050\,788\,u - 234.040\,952\,u - 4.002\,603\,u = 7.233 \times 10^{-3}\,u$	Calculate the mass change.	1 mark
$E = \Delta mc^2$	Relate energy to mass change.	1 mark
$E = 7.233 \times 10^{-3}\,u \times 1.661 \times 10^{-27}\,kg\,u^1 \times (2.998 \times 10^8\,m\,s^{-1})^2 = 1.08 \times 10^{-12}\,J$	Calculate energy, Remembering to convert from u to kg.	1 mark

Question set 6.5

1 **a** $m = \gamma m_o$

b $p = \gamma p_o$

c $E = m_o c^2$

d $\Delta E = \Delta m_o c^2$

2 The mass defect is the change in mass during a nuclear process, which is converted into energy at the rate $E = \Delta mc^2$.

3 The mass defect determines how much energy is released in a spontaneous process, or how much energy must be provided to get a reaction to occur.

4 The kinetic energy is:

$E = (\gamma - 1)m_p c^2 = 1.0 \times 10^{-10}\,J$

5 $E = (\gamma - 1)mc^2$

$$\gamma = \frac{E}{mc^2} + 1 = 2.17$$

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\text{So } v = c\sqrt{1 - \frac{1}{\gamma^2}} = 0.89\,c$$

6 $\Delta m = 213.995\,u - 209.984\,u - 4.003\,u = 8.00 \times 10^{-3}\,u$

$E = \Delta mc^2 = 8.0 \times 10^{-3}\,u \times 1.661 \times 10^{-27}\,kg\,u^{-1} \times (2.998 \times 10^8\,m\,s^{-1})^2 = 1.19 \times 10^{-12}\,J$

$E = 7.46\,MeV$

7 10^{-12} J is a typical order of magnitude energy released in a decay reaction.

190 kg of U–235 has around $\frac{190\ \text{kg}}{235 \times 1.6 \times 10^{-27}\ \text{kg}} = 5 \times 10^{26}$ atoms.

So if they all decayed you would get around 5×10^{14} J, or order of magnitude 10^{15} J

8 Students' answers will vary.

9 Students' answers will vary.

Chapter review questions

1 The laws of physics are the same in all inertial frames.

The speed of light, c, is constant in all inertial reference frames.

2 a A non-accelerating frame of reference.

b Movement relative to a particular object, medium or frame of reference.

c Time measured in a stationary frame of reference.

d Motion described by the proper time and length.

e Time appears slower in a reference frame that is moving relative to a stationary frame.

f Length appears shorter in a reference frame that is moving relative to a stationary frame.

g Difference between total mass before and after a nuclear decay or reaction.

3 $x = x' + v\Delta t$

$y = y' + v\Delta t$

$\vec{v}_{AB} = \vec{v}_{AC} + \vec{v}_{CB}$

$$t = \frac{t_o}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\gamma = \frac{1}{\sqrt{1 - \beta^2}}$$

$$\beta = \frac{v}{c}$$

$$L = \frac{L_o}{\gamma}$$

$$m = \frac{m_o}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$m = \gamma m_o$

$$p = \frac{m_o v_o}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$p = \frac{p_o}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$p = \gamma p_o$$

$$E = mc^2$$

$$E = m_o c^2$$

$$\lambda = \lambda_o \sqrt{\frac{1 - \beta}{1 + \beta}}$$

4 **a** It unifies the electric and magnetic fields, and describes the relationship between them.

b It predicts that the light does not need a medium and has a constant speed in vacuum.

5 **a** $v_p = v_{\text{p to train}} + v_{\text{train}} = (16\,\text{m}\,\text{s}^{-1} - 3\,\text{m}\,\text{s}^{-1}) = 13\,\text{m}\,\text{s}^{-1}$

b $s = vt = 13\,\text{m}\,\text{s}^{-1} \times 5\,\text{s} = 65\,\text{m}$

6 Michelson and Morley used the interference of light which had been passed through perpendicular paths of the same length to create an interference pattern. A shift in the pattern due to a difference in the speed of the light in the two paths would have provided evidence for the aether. No such shift was observed (a null result) indicating that there was no aether.

7 Simultaneity is the occurrence of two events at the different positions but at the same time. Relativistically, two events that are simultaneous in one reference frame will not be simultaneous in a second reference frame moving relative to the first frame. For example, a train moving into a tunnel such that an outside observer sees the front of the train reach the end of the tunnel simultaneously with the end of the train reaching the beginning of the tunnel. These events are not simultaneous in the frame of an observer on the train.

8 **a** **i** Light has a speed of $c = 3.00 \times 10^8\,\text{m}\,\text{s}^{-1}$ in vacuum for all observers.

ii Light has a speed of $c = 3.00 \times 10^8\,\text{m}\,\text{s}^{-1}$ in vacuum for all observers.

$$L = \frac{L_o}{\gamma}$$

$$\Rightarrow L_o = L \times \gamma$$

$$\Rightarrow L_o = \frac{L}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\Rightarrow L_o = \frac{1.8 \times 10^9\ \text{m}}{\sqrt{1 - \left(\frac{0.6c}{c}\right)^2}}$$

b i $\Rightarrow L_o = \dfrac{1.8 \times 10^9 \text{ m}}{0.8}$

$\Rightarrow L_o = 2.25 \times 10^9 \text{ m}$

$\text{time} = \dfrac{\text{distance}}{\text{velocity}}$

$\Rightarrow \text{time} = \dfrac{2.25 \times 10^9 \text{ m}}{3.0 \times 10^8 \text{ m s}^{-1}}$

$\Rightarrow \text{time} = 7.5 \text{ s}$

$t = \dfrac{t_o}{\sqrt{1 - \dfrac{v^2}{c^2}}}$

$\Rightarrow t = \dfrac{2.0 \text{ s}}{\sqrt{1 - 0.6^2}}$

ii $\Rightarrow t = 2.5 \text{ s}$

$m = \dfrac{m_o}{\sqrt{1 - \dfrac{v^2}{c^2}}}$

$1.005\, m_o = \dfrac{m_o}{\sqrt{1 - \dfrac{v^2}{c^2}}}$

$\Rightarrow \sqrt{1 - \dfrac{v^2}{c^2}} = \dfrac{1}{1.005} = 0.9950$

$\Rightarrow 1 - \dfrac{v^2}{c^2} = 0.9901$

$\Rightarrow v^2 = 0.0099\, c^2$

9 $\Rightarrow v = 0.10\, c$

$49 \text{ years} = t = \dfrac{t}{\sqrt{1 - \dfrac{0.7^2}{c^2}}}$

$\Rightarrow t = 49 \times \sqrt{1 - \dfrac{0.7^2}{c^2}}$

10 $\Rightarrow t = 35 \text{ years}$

11 $t_{\text{pion}} = \dfrac{t_{\text{Earth}}}{\gamma} = t_{\text{Earth}} \sqrt{1 - \dfrac{v^2}{c^2}} = 26 \text{ ns} \sqrt{1 - 0.5^2} = 23 \text{ ns}$

12 $L = \frac{L_o}{\gamma} = L_o\sqrt{1 - \frac{v^2}{c^2}}$

$\Rightarrow L = 10000 \text{ m} \times \sqrt{1 - \frac{0.999c^2}{c^2}}$

$\Rightarrow L = 447 \text{ m}$

13 a $4m_Hc^2 = 4(1.673 \times 10^{-27} \text{ kg})(3.0 \times 10^8 \text{ m s}^{-1})^2$

$\Rightarrow 4m_Hc^2 = 6.022\,80 \times 10^{-10} \text{ J}$

$m_{He}c^2 = (6.644 \times 10^{-27} \text{ kg})(3.0 \times 10^8 \text{ m s}^{-1})^2$

$\Rightarrow m_{He}c^2 = 5.9796 \times 10^{-10} \text{ J}$

$2m_ec^2 = 2(9.1 \times 10^{-31} \text{ kg})(3.0 \times 10^8 \text{ m s}^{-1})^2$

$\Rightarrow 2m_ec^2 = 0.001\,638 \times 10^{-10} \text{ J}$

$\Delta E = 4m_Hc^2 - m_{He}c^2 - 2m_ec^2$

$\Rightarrow \Delta E = 4.2 \times 10^{-12} \text{ J}$

b Four hydrogen atoms have a mass of:

$4 \times 1.673 \times 10^{-27}\text{kg}$

Number of four atoms of hydrogen in 10 kg:

$= \frac{10 \text{ kg}}{4 \times 1.673 \times 10^{-27} \text{ kg}}$

$= 1.494 \times 10^{27}$

Total energy from the fusion of 10 kg of four hydrogen atoms:

$= 1.494 \times 10^{27} \times 4.2 \times 10^{-12}\text{ J}$

$= 6.3 \times 10^{15}\text{J}$

14 $t = t_o\gamma$

$\Rightarrow t_o = \frac{t}{\gamma}$

$\Rightarrow t_o = 20\sqrt{1 - 0.6^2}$

$\Rightarrow t_o = 16 \text{ years}$

15 a Because their trips were identical mathematically, they will be the same age.

b Time away by each twin:

$$\frac{\text{distance}}{\text{velocity}} = \frac{14 \text{ light-years}}{0.61\ c}$$

$$= \frac{14 \text{ years} \times c}{0.61\ c} = 23 \text{ years}$$

$$t_o = t\sqrt{1 - \frac{v^2}{c^2}}$$

$$\Rightarrow t_o = 23\sqrt{1 - 0.61^2}$$

$$\Rightarrow t_o = 18 \text{ years}$$

Ignatius will appear 5 years older than Xaviar and Maxine.

16 The pilot of a non-accelerating spacecraft, moving away from Earth at great speed, celebrates the passing of six birthdays. Earth-bound observers measure this elapsed time to be 10 years. Relative to Earth:

a $t = \dfrac{t_o}{\sqrt{1 - \frac{v^2}{c^2}}}$

$$\Rightarrow \sqrt{1 - \frac{v^2}{c^2}} = \frac{t_o}{t} = \frac{6}{10} = 0.6$$

$$v = c\sqrt{1 - 0.6^2} = 0.8\ c$$

b Distance $=$ velocity $(v) \times$ time (t)

$\Rightarrow$ Distance $= 0.8\ c \times 10$ years

$\Rightarrow$ Distance $= 0.8$ light-years

17 a $v = \dfrac{\Delta s}{\Delta t} = \dfrac{85 \text{ m}}{0.377\ \mu\text{s}}$

$$= 2.25 \times 10^8 \text{ m s}^{-1}$$

$$= 0.75\ c$$

b $L = \dfrac{L_o}{\sqrt{1 - \frac{v^2}{c^2}}}$

$$\Rightarrow L_o = L\sqrt{1 - \frac{v^2}{c^2}}$$

$$\Rightarrow L_o = 85 \text{ m}\sqrt{1 - 0.75^2}$$

$$\Rightarrow L_o = 56 \text{ m}$$

18 Ignoring any kinetic energy of the electrons:

$$E = 2m_oc^2 = 2 \times 9.109 \times 10^{-31} \text{kg} \times (2.998 \times 10^8 \text{m s}^{-1})^2 = 1.637 \times 10^{-13} \text{J}$$

19 a **i** $s = vt = 0.9995\,c \times 2.2 \times 10^{-6}\,\text{s} = 660\,\text{m}$

b $\text{L} = \dfrac{L_o}{\gamma} = \text{L}_o\sqrt{1 - \dfrac{v^2}{c^2}} = 5000\,\text{m}\sqrt{1 - 0.995^2} = 500\,\text{m}$

c Yes, those that have a velocity in the right direction are likely to reach the surface of Earth.

20 See Figure 6.15. As v increases towards c, γ increases very rapidly, towards infinite. Remember that $\gamma = \dfrac{1}{\sqrt{1 - \dfrac{v^2}{c^2}}}$, so in the limit $v = c$, $\gamma = \infty$. Kinetic energy is proportional to γ, so as $v \rightarrow c$, $KE \rightarrow \infty$.

As all values of v up to c are allowed, all values of KE from zero to infinite are allowed.

21 a $p = \gamma p_o = \gamma m_o v = \dfrac{m_o v}{\sqrt{1 - \dfrac{v^2}{c^2}}}$

$$= \frac{9.1 \times 10^{-31}\text{ kg} \times 0.9996 \times 3.0 \times 10^{8}\text{ m s}^{-1}}{\sqrt{1 - 0.9996^2}} = 9.65 \times 10^{-21}\text{ kg ms}^{-1}$$

b From Chapter 4: $p = mv = qBr$

$$\text{r}_o = \frac{p}{qB} = \frac{9.65 \times 10^{-21}\text{ kg m s}^{-1}}{1.6 \times 10^{-19} C \times 2.8\ T} = 0.021\text{ m in the electron's frame}$$

In Earth's frame:

$$r = \gamma \div r_o = \frac{r_o}{\sqrt{1 - \dfrac{v^2}{c^2}}} = \frac{0.021\text{ m}}{\sqrt{1 - 0.9996^2}} = 0.74\,\text{m}$$

Reflecting

22 Students' answers will vary.